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Airplane Performance Sensitivities to Lateral and Vertical Profiles

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Airplane performance sensitivity to the choice of lateral and vertical profiles is studied based on data recorded on in-service flights of a 727-200 airplane equipped with JT9D-7 engines. Lateral profile performance sensitivity is based on analysis of flight technical or guidance error and on varying degress of direct flight clearance. Vertical analysis is based on an aerodynamic simulation of the recorded flights. Sensitivity to the choice of vertical profile is studied by simulating the actual profile that was flown, two perfectly flown constant IAS/Mach profiles, and minimum-cost and minimum-fuel optimum profiles. Some of the practical considerations in implementing the real-time optimum profile algorithms and some of the difficulties encountered with the optimum profile computations are discussed. Predicted fuel and time savings are related to the air traffic control (ATC) environment experienced by the airlines to obtain net savings predictions with varying degrees of ATC restriction.

Nomenclature

D = drag
$\dot{E}$ = rate of energy change
F_N = thrust
H_T = true altitude
H_{cr} = cruise altitude
p = pressure
R_G = gas constant
T = absolute temperature
V = true airspeed
V_{cr} = cruise true airspeed
V_W = tailwind speed
W = aircraft gross weight
W_F = fuel flow
γ = flight-path angle
δ = scale factor (weight per unit time)
ΔE = change in energy
Δt = time for energy transition
σ = cost index

Introduction

With fuel costs escalating, the air transportation industry is investigating methods for flying an airplane to minimize fuel costs or direct operating costs. Two methods considered are modifying (reducing) the aircraft speed as published in the pilots' handbooks and flying "optimal" control speed and thrust schedules. The latter are derived from extensions of the energy formulation for aircraft performance problems first described by Rutowski.[1] Rutowski's technique was applicable to fixed-throttle, minimum-time-to-climb, and minimum-fuel-to-altitude problems. Thomas and Porter[2] extended the method to fixed-range conditions with variable throttle controls and mixed time and fuel cost functions.

In the formulations, the aircraft dynamics are approximated by a second-order equation set. Dynamics on flight-path angle are explicitly ignored. The resulting optimal control problem reduces to finding the extremum of a particular function for climb, cruise, and descent. Schultz and Zagalsky[3] noted that depending on the form of the equation set, the maximum principle does not always yield optimum performance.

Stengel and Marcus, in their minimum-fuel investigations, explicitly included flight-path angle dynamics.[4] This optimal control formulation required solving a two-point boundary value problem with a gradient technique. While the formulation is more exact, the gradient solution cannot be implemented in a real-time avionics computer; the reduced states formulations, requiring functional minimization, can be solved in real time. Hence, of the two, only the latter is acceptable for on-line implementation. Calise[5] analyzed the more general problem using concepts taken from singular perturbation theory. While fast dynamics (such as flight-path angle) are implicitly ignored in the reduced-order formulation, they can be accounted for by boundary value analysis. Moreover, in a comparison of the fuel used with controls generated by the gradient technique vs the singular perturbation technique, case studies involving jet transport aircraft revealed that the results were essentially equivalent; the "optimal" gradient-derived solution required 4 lb more fuel (out of 7416 lb) on a 200 n.mi. flight and 18 lb less fuel (out of 15,266) on a 500 n.mi. flight compared to the reduced-order "optimal" singular perturbation solution. Algorithms for real-time avionics computers have been developed utilizing the reduced state energy formulation.[6] Recent research has focused on adding more mathematical rigor to the optimal controls developed under the reduced-order formulation.[7]

This paper addresses potential savings which may be encountered by airline users in actual revenue service. Optimal controls, defined to be those generated by a reduced-order (singular perturbation) formulation are compared against conventional airline handbook profiles and minimum-fuel constant IAS/Mach or "handbook-type" profiles. The conventional airline profiles can be thought of as minimizing direct operating cost. The simulated conditions consist of the actual physical and air traffic control (ATC) environment encountered by airline flights and an aircraft model which

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includes flight-path and autopilot dynamics. Previously reported simulations did not have to contend with real varying winds, temperatures, or ATC constraints[8]; these constraints tend to reduce the benefits for flying optimal trajectories. This study is meant to determine the actual benefits to be expected when flying the more difficult to control, monitor, and compute "optimal controls" formulated over the past decade vs more conventional "handbook profiles."

In all, about 600 flights of a 727 airplane equipped with JT9D-7 engines were recorded with 40 data parameters being sampled for each second of flight. In addition, peripheral information relating to each flight was gathered by hand and later merged with the data recorded in flight. Two hundred flights have been reduced to a more convenient form for analysis and stored on magnetic tapes.

A unique element of the instrumentation was scanning distance measuring equipment (DME) specially prepared for this program. This device was capable of sequentially and continuously searching through all possible DME channels in rapid succession, outputting to the recorder the distance and channel number of all DME's located. This information was then processed in a ground-based computer to yield an accurate record of the track of the flight, with related byproducts such as ground speed and wind parameters.

The reduced data tapes actually contain 49 parameters per second of flight; the additional data being added after ground processing. The data exists in several categories: performance, environment, position and attitude, configuration, and header (one per flight). A sufficient number of data parameters are recorded to reproduce a complete history of the airplane flight path and performance.

Lateral Analysis

Two studies were undertaken to search for potential savings which could have been realized on specific airline flights through application of the principles of area navigation; that is, through better management of the lateral track.

The first study was designed to illuminate two potential sources for savings in distance flown. One of these involves "Flight Technical Error," with the question being: "How much distance could be saved if flight technical error (wandering with respect to the center line of the intended route or track) could be reduced to zero?" The potential for savings arises from the fact that most flights do not operate with the autopilot coupled to the radio navigation signals which identify the track centerline, due to vagaries in the signal itself.

The other potential source of savings investigated was that available from flying a greater portion of great circle direct routes, as opposed to airways. The goal here was to assess the sensitivity of the saving in trip distance to the length of the direct segment. For this purpose, each trip was evaluated to determine the effect of flying direct between the following points or over the following segments:

1) Above 18,000 ft.
2) Above 10,000 ft.
3) From the point where the flight could have proceeded without radar guidance, to an initial approach fix or equivalent in the destination terminal area. This is assumed to be the best practical direct route and is referred to in this way later.
4) From the point where the aircraft was first turned toward its destination, to the point where it turned final. This is considered the best possible direct route (but not very practical) and is referred to in this way later.
5) From takeoff point to touchdown point. This is examined for reference only.

For the purpose of the analysis, the actual flight of the aircraft is assumed to be identical to the recorded path outside the above segments.

The results of the study are given in Table 1 and are expressed as the percentage saving in total trip ground miles.

Table 1 Reduction in distance flown

Source of distance savings	Ground distance saved, %	Fuel saved, %	Time saved, %
Elimination of flight technical error	0.8	0.6	0.7
Direct routing:			
1) Above 18,000 ft	1.9	1.5	1.6
2) Above 10,000 ft	2.6	2.1	2.2
3) Best practical	3.0	2.4	2.6
4) Best possible	4.6	3.6	3.9
5) Takeoff to touchdown	8.0	6.3	6.8

These results were obtained from a 20-flight sample which was statistically similar to the 200-flight data base. It can be shown that a 20-flight sample, while small, yields cumulative averages that are quite stable. The average flight was 427 n.mi.

In order to facilitate the addition of these lateral savings to those confirmed in the vertical analysis described in the next section, it was assumed that these lateral distance savings would have the effect of reducing the cruise phase of flight only. Since the ratios of fuel and time per unit distance flown are different in cruise than for the flight as a whole, the above distance savings have to be weighted in order to arrive at an estimate of fuel and time savings. The weighting factors used were 0.79 for fuel and 0.85 for time, which are percentages of fuel and time consumed in the cruise phase per percentage distance flown in the cruise phase. The application of these factors to the distance savings result in computed fuel and time savings as shown in Table 1.

A second study analyzed the lateral profiles of 98 flights with a total flight distance of over 65,000 mi. The analysis concentrated on the flight technical error and comparison of actual distance flown with flight plan distance and great circle distance. Comparisons were based both on ground and air miles. Flight technical error was found to be 1.0% of ground distance or roughly similar to the data in Table 1.

Vertical Analysis

Fuel and cost savings can be realized by proper choice of the vertical profile. The real questions are how much can be saved and how complex is the profile necessary to achieve the savings. These questions are investigated through simulation analyses of the recorded data to predict performance sensitivities.

Simulator

The simulator uses the point mass differential equations of aircraft motion. Thrust control and pitch control laws are included in the simulator to provide steering to the desired profile. Heading and bank angles of the simulator are slaved to follow recorded heading and bank angles. Drag coefficient is a tabulated function of lift coefficient and Mach. Thrust and fuel flow are tabulated functions of engine pressure ratio (EPR), Mach, and altitude. The differential equations are numerically integrated. Drag, thrust, and fuel flow values are interpolated from the tabular data.

The simulated environment is described by temperature, pressure, true altitude, and wind velocity. Each of these quantities is related to values recorded along the actual trajectory with corrections for deviations between the simulated trajectory and the actual trajectory. True altitude is estimated by integrating the differential relationship $dH_T = (R_G T/p)\,dp$. Wind speed was derived from scanning DME measurements and recorded true airspeed. Several different vertical profiles are simulated with each recorded flight segment. Each profile simulation uses the same environment and lateral path and is only performed over that portion of the flight in which the airplane is in a clean configuration.

The simulator was validated by comparing simulation results to the recorded performance of the actual airplane. Total fuel and total time computed by the simulated actual flight agreed to within 3% of the measured total fuel and time. The analyses presented here predict performance sensitivities rather than absolute performance. Sensitivity analysis is not strongly affected by small errors in modeling the real world.

Profiles Analyzed

Simulation results are presented for three types of vertical profiles—the actual profile, constant IAS/Mach profiles, and optimum profiles. The cruise altitude used in the actual profile was also used in the other two types of profiles. All profiles have a 250 knot IAS constraint below 10,000 ft. All profiles begin with a 250 knot IAS initial velocity.

The actual profile uses recorded EPR and true airspeed as target values for the simulator control laws during climb phase. It uses recorded altitude and true airspeed in the cruise and descent phases. A geographically constrained descent is used to assure that the simulation terminates at the destination target.

Two constant IAS/Mach profiles are simulated. One profile obtained from an airline operator's handbook can be described as follows: "At 10,000 ft accelerate to 340 knots indicated airspeed (IAS) and hold this speed in climb until Mach 0.78 is reached (appr. 25,000 ft). Climb at Mach 0.78 to cruise altitude. Cruise at Mach 0.8. Reverse the procedure in descent except hold Mach 0.8 initially."

A second constant IAS/Mach profile was compiled by the author to approximately minimize fuel required. This profile is analogous to the airline handbook profile with different speed values. It uses a weight-dependent climb schedule recommended by the airframe manufacturer which varies from 280 knots IAS/0.78 Mach for 130,000-lb aircraft to 295 knots IAS/0.78 Mach for 160,000-lb aircraft.[9] Maximum range cruise velocity is used which is obtained from the airplane's specific range curves. Descent schedule is 240 knots IAS/0.78 Mach. The descent schedule was chosen based on experience with the optimum descent profile.

Both constant IAS/Mach profiles use maximum climb rated thrust in climb phase and flight idle thrust in descent phase. Descent guidance indirectly references destination through choice of top-of-descent location. For purposes of analysis, top-of-descent location is chosen to make the descent profile cross destination altitude at destination range. This descent profile is almost impossible to realize in practice since temperature and wind variation during descent will move the aircraft off of its predicted path.

The optimum profile is based on cost of the total flight, total fuel used on the flight, or total time of flight.[10] Cost index, σ, is the weighting factor that defines cost as a weighted sum of total fuel used and total time of flight. The cost parameter used here is expressed as the amount of fuel whose value is equivalent to the total cost:

$$\text{Cost} = \frac{\sigma\delta}{1-\sigma}(\text{total time of flight}) + (\text{fuel used}) \tag{1}$$

where δ is a constant parameter to make the equation have consistent units since σ is a dimensionless parameter. The analysis presented here assumes that $\delta = 1$ lb/s. Cost index can be related to the airline's time- and fuel-dependent costs:

$$\sigma = \frac{\text{time cost (\$/s)}}{\text{time cost (\$/s)} + \delta \text{ fuel cost (\$/lb)}} \tag{2}$$

Minimum-fuel trajectory corresponds to $\sigma = 0$ and minimum-time trajectory corresponds to $\sigma = 1$. The minimum cost corresponds to a value defined by Eq. (2). It has been assumed in this paper that minimum cost corresponds to $\sigma = 0.7$. This gives performance comparable to the airline handbook. Profiles are simulated for each flight segment analyzed with σ values of 0 and 0.7.

Cost as defined by Eq. (1) has units of pounds of fuel. This is the cost presented throughout this report. Cost may be converted to dollars by multiplying by fuel cost in dollars per pound.

A later section describes the optimum profile in more detail.

Time-Fuel Plots

Performance with all possible vertical profiles for one specific lateral profile, environment, and aircraft weight can be shown on a single graph by plotting total fuel used against total time required. Each vertical profile is represented by a point on the fuel vs time graph. The optimum profiles for all values of cost index between 0 and 1 appear as a locus of points or a curve on the time-fuel graph. Profiles with equal cost appear as a locus of points that forms a constant cost grid. The constant cost grid lines are parallel straight lines. The slope is dependent on the cost index. This concept is also used in Ref. 10.

The time-fuel plot for an 803 n.mi. flight is shown in Fig. 1. Optimum profiles are shown for all cost indices. Both constant IAS/Mach profiles and the actual profile simulation are shown. The constant cost grid is shown for a 0.7 cost index. The perfectly flown airline handbook is equivalent to an optimum profile with cost index near 0.7. The perfectly flown minimum fuel constant IAS/Mach profile requires very nearly the same fuel as the minimum fuel optimum profile.

The performance of the actual flight is not optimum for any cost index. It requires 229 additional pounds of fuel for essentially the same time of flight as the airline handbook. When compared to the minimum fuel profile, the actual flight has a large fuel penalty. This may not be a valid comparison because the airlines do not fly minimum fuel profiles, they attempt to fly profiles which minimize direct operating costs (fuel and time) and satisfy other constraints (scheduling). The comparison criteria must be established by airline policy; i.e., a cost index value must be chosen. A particular airline's costs may not correspond to a 0.7 cost index.

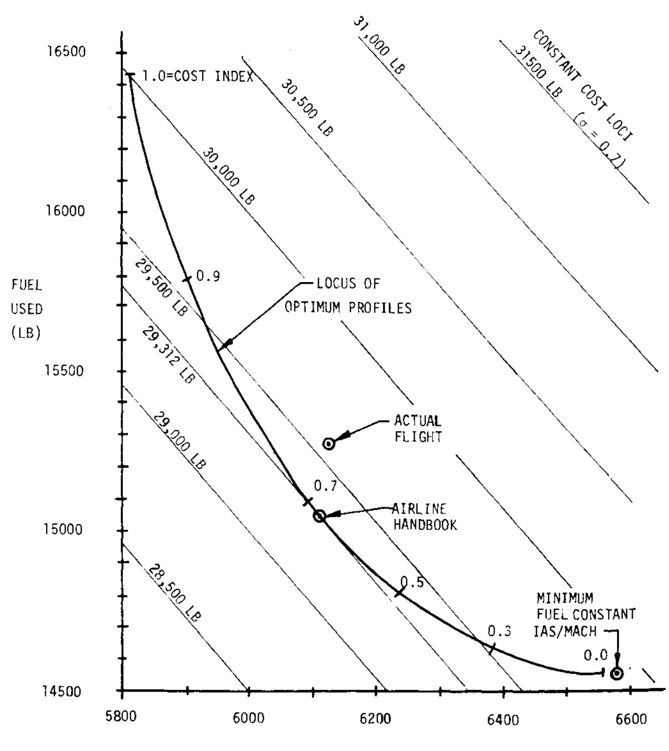

Fig. 1 Time vs fuel plot of typical flight (803 n.mi. range).

Table 2 Actual profile and minimum-cost profile simulation results (cost index = 0.7)

Flight length, n.mi.	Actual profile			Airline handbook profile			Minimum-cost optimum profile		
	Fuel, lb	Time, s	Cost, lb	Fuel, lb	Time, s	Cost, lb	Fuel, lb	Time, s	Cost, lb
390	8,382	3,048	15,494	8,429	2,945	15,301	8413	2934	15,259
287	6,164	2,053	10,954	6,075	2,045	10,847	5,965	1,980	10,585
579	11,068	4,083	20,595	10,749	4,013	20,113	10,719	4,052	20,174
383	7,656	2,295	13,011	7,590	2,263	12,870	7,481	2,299	12,845
472	8,573	3,610	16,996	8,145	3,520	16,358	8,107	3,515	16,309
104	2,956	814	4,855	3,050	801	,4919	3,168	777	4,981
803	15,275	6,126	29,569	15,046	6,102	29,284	15,093	6,094	29,312
790	16,195	6,175	30,603	16,045	6,103	30,285	16,100	6,143	30,434
96	5,647	1,874	10,020	5,548	1,848	9,860	5,572	1,814	9,805
863	14,762	5,816	28,333	14,614	5,761	28,056	14,592	5,767	28,048
Average: 477	9,668	3,589	18,045	9,529	3,540	17,789	9,521	3,538	17,776
Saving relative to actual profile:	...	...	...	139 (1.3%)	49 (1.2%)	256 (1.2%)	147 (1.4%)	51 (1.3%)	269 (1.3%)

The cost index may also be viewed as an optimization parameter to compute the minimum fuel profile with a time constraint. It can be seen from Fig. 1 how a specified time of flight determines the cost index for the constrained minimum fuel profile. This concept can be used to establish a suitable cost index value for an airline. Target flight times are chosen for a few representative flights and the cost index to meet the time constraint is found by an iterative process of simulating the flights on a ground-based computer for several values of cost index. The selected cost index is the value that best meets the schedule requirements. This is done without explicit knowledge of the airline's time cost sensitivity.

Summary of Results

Ten flight segments have been simulated. The performance of the airplane as it was actually flown has been compared to the performance it would have achieved if it had been flown according to the airline handbook and minimum cost optimum profile as shown in Table 2. Data are summarized in the bottom row of the table as the average performance of the ten flights. Both the airline handbook and minimum cost optimum profiles show cost savings equivalent to approximately 260 lb of fuel for the average flight. This savings is realized while simultaneously realizing a decrease in flight time of approximately 50 s. The percentage savings shown in Table 2 are based on total flight fuel and time rather than only the fuel and time used over the simulated portion of the flight.

Three flight segments analyzed have apparent ATC altitude restrictions on descent. These segments are characterized by short cruise segments in the middle of descent. Two flight segments have 250 knots IAS velocity restrictions above 10,000 ft in descent. The handbook and optimum profile simulations of Table 2 honor these altitude restrictions but not the velocity restrictions. The airline handbook simulations were rerun with the velocity restrictions to quantify these effects. Cost of the two flight segments increased by 0.8% and 0.3%, respectively, due to the velocity restriction.

Performance along the ten flight segments has been simulated with the minimum fuel handbook profile and the minimum fuel optimum profile as shown in Table 3. These simulations did not honor the apparent ATC restrictions. Fuel usage on the two profiles is nearly the same.

It is apparent from these results that performance from the airline handbook profile or minimum fuel constant IAS/Mach is approximately equal to that of the optimum profile at an equivalent cost index. This conclusion must be qualified by two observations—the two constant IAS/Mach profiles are perfectly flown and the optimum profile is not truly optimum. A perfectly flown constant IAS/Mach profile

Table 3 Minimum-fuel profile simulation results

Flight length, n.mi.	Minimum-fuel constant IAS/ Mach profile		Minimum-fuel optimum profile	
	Fuel, lb	Time, s	Fuel, lb	Time, s
390	7,533	3,152	7,579	3,177
287	5,294	2,313	5,296	2,353
579	9,895	4,500	9,882	4,643
383	6,897	2,555	6,882	2,638
472	7,645	3,861	7,643	4,009
104	2,766	1,005	2,788	976
803	14,597	6,576	14,555	6,562
790	15,400	6,658	15,463	6,654
96	4,989	2,215	5,053	2,145
863	13,869	6,272	13,847	6,381
Average: 477	8,889	3,911	8,899	3,954

requires precise tracking of the velocity and throttle schedules and top-of-descent computation that allows the descent to reach destination. It is unlikely that a completely manually flown profile can achieve the necessary precision, particularly in descent.

The profile optimization computations (as defined by the reduced-order formulation) are suboptimal, as evidenced by the fact that performance from the handbook profiles are slightly better than that from the optimum profiles for some of the flight segments analyzed. The optimum profile and reasons for not achieving a true optimum are discussed further in a later section.

The actual profile does have a significant performance penalty. It is not obvious what characteristic of the actual profile causes the degraded performance. In most cases the airline handbook is followed reasonably closely. Additional analysis on one flight segment has been performed to illuminate this difference. Simulation of the actual profile uses recorded velocity and engine pressure ratio (EPR) as climb target values and recorded velocity and altitude as descent target values. Table 4 shows several variations on these profiles. Fuel and time for climb, cruise, and descent phases are computed for fixed ranges along the path so the results can be compared without compensating for top-of-climb or top-of-descent location. The actual profile is repeated with EPR constrained to maximum climb rating during climb and flight idle during descent. Recorded descent altitude profile is not used. If the top-of-descent (TOD) remains at the recorded location, descent at flight idle is too

Table 4 Profile sensitivities for a selected flight (579 n.mi. range)

Profile	Fuel used, lb Climb	Fuel used, lb Cruise	Fuel used, lb Descent	Fuel used, lb Total	Time, s Climb	Time, s Cruise	Time, s Descent	Time, s Total	Cost ($\sigma = 0.7$)
Actual profile	5,722	3,479	1,882	11,083	1,458	1,388	1,239	4,085	20,615
Modified actual profile Max climb EPR Flight idle EPR descent Actual TOD	5,702	3,479	20.65	11,246	1,445	1,389	1,384	4,218	21,088
Modified actual profile Max climb EPR Flight idle EPR descent Optimum TOD	5,702	3,479	1,638	10,819	1,445	1,389	1,183	4,017	20,192
Airline handbook profile	5,689	3,467	1,603	10.759	1,444	1,393	1,172	4,009	20,113
Modified airline handbook profile 3 mile/1000 ft descent Ddescent velocity schedule	5,689	3,467	1,734	10,890	1,444	1,393	1,113	3,950	20,107

Table 5 Total performance savings

a) Scenario composition

Scenario experience level	Mix of direct flights, % None	Above 18,000 ft	Above 10,000 ft	Best	Realizable vertical savings, % Climb	Cruise	Descent
Low	50	20	15	15	85	100	65
Medium	20	35	20	25	90	100	75
High	10	15	25	50	95	100	85

b) Time and fuel savings

Scenario experience level	Lateral savings, % Fuel	Lateral savings, % Time	Lateral and vertical savings, % Fuel	Lateral and vertical savings, % Time	Total savings converted to fuel savings, %
Low	1.3	1.4	2.4	2.2	3.8
Medium	1.7	1.8	2.8	2.7	4.5
High	2.0	2.2	3.2	3.2	5.3

steep and a cruise phase must be inserted at low altitude. This creates a large fuel penalty (11,246 vs 11,083 lb). If the correct TOD is computed, the performance with recorded velocity profile and maximum climb rating/flight idle EPR profiles is very close to the handbook performance. The results also demonstrated that TOD must be properly computed to allow a flight idle EPR descent profile. Another simulation is performed using a handbook profile with a straight line descent path. The descent path has a slope of 3 n.mi./1000 ft. Descent EPR is computed to maintain the descent path and the velocity profile. The required EPR settings are within 10% of flight idle EPR. This profile has a significant fuel penalty, but not as great as the profile with the wrong TOD and low-altitude cruise segment. However, it is faster and yields the lowest cost.

It may be concluded that the greatest performance penalty is due to nonoptimum throttle management in descent combined with an improper choice of TOD that does not allow the throttle to remain at or near flight idle. The recorded EPR settings on all flights generally held a flight idle setting. However, the recorded EPR settings were occasionally brought off of flight idle to hold a descent path and speed schedule..

The geographic descent control law used in the optimum profile simulations solves most of the problems of descent. It requires a precise reference descent path computed for the aerodynamic characteristics of the aircraft, winds, temperature, etc. The descent path is computed by integrating the equations of motion backwards in time from destination using an assumed aircraft weight at destination. Unexpected conditions in descent will require some throttle activity. Descent target is met with a minimum of throttle activity at the low altitudes.

Composite Lateral and Vertical Analysis

Lateral and vertical profiles have been analyzed to determine potential fuel, time, and cost savings when compared to the profiles actually flown. Potential lateral savings are closely tied to the ability to fly direct. Potential vertical savings for the optimum profile and the equivalent perfectly flown constant IAS/Mach profile are nearly equal. The best vertical profile (optimum or constant IAS/Mach) depends on the cost index which specifies the tradeoff between fuel and time minimization. Table 5 summarizes the composite savings from both lateral and vertical profile. To obtain realistic estimates of the realizable savings, it is necessary to assume a mix of direct flights and airway flights which depends on the amount of air traffic control (ATC) clearance that can be obtained and on other constraints such as weather. It is also necessary to assume that not all of the vertical savings can be realized due to ATC constraints. Table 5a presents three scenarios called low, medium, and high experience levels which define the assumed probability of being able to obtain the desired clearances. Table 5b shows the corresponding fuel and time savings that would be obtained with these three scenarios while continuing to operate at the 0.7 cost index.

A short-term objective of an airline may be to minimize fuel while maintaining the existing schedule. Therefore, the vertical profile cost index can be reduced just enough to cancel the time savings, resulting in additional fuel savings. The sensitivity of fuel and time savings to cost index is

$$\frac{\partial \text{ Fuel}}{\partial \text{ Time}} = \frac{\sigma\delta}{1-\sigma} \tag{3}$$

for small changes in cost index σ. For time changes up to a 3.2% time increase, a more conservative value for the sensitivity at $\sigma = 0.7$ is

$$\frac{\Delta \text{ Fuel}}{\Delta \text{ Time}} = -1.7 \text{ lb/s} \tag{4}$$

which is obtained from Fig. 1. The last column in Table 5b gives the results of converting time savings to fuel savings as described above.

Optimal Control Computation

The optimum control strategy is based on the reduced energy formulation described earlier. In this formulation the optimal control problem reduces to minimizing or maximizing certain well-behaved functions along the trajectory. The cruise solution is first found after which the climb and descent functions can be defined. The optimal controls are found by searching velocity and thrust in climb and descent and velocity in cruise (as thrust is set equal to drag for the cruise velocity search). Tabular or polynomial representations of thrust and fuel flow vs a parameter such as engine pressure ratio, engine rpm, or % power and drag vs lift are required to complete the data package.

The optimum cruise velocity, V_{cr}^{opt}, and altitude, H_{cr}^{opt}, are found as follows:

$$V_{cr}^{opt}, H_{cr}^{opt} = \text{arguments which maximize}\left[\frac{V+V_w}{\sigma+(1-\sigma)W_F}\right]_{\substack{F_N=D\\ \gamma=0}} \tag{5}$$

Define the optimal cruise performance (CP) as follows:

$$\mathrm{CP}^{opt} = \max\left[\frac{V_{cr}+V_w}{\sigma+(1-\sigma)W_F}\right]_{\substack{F_N=D\\ \gamma=0}} \tag{6}$$

Then the control variables F_N and V, which maximize

$$\left[\frac{(V+V_w)-\mathrm{CP}^{opt}(\sigma+(1-\sigma)W_F)}{(F_N-D)(V+V_w)/W}\right]_{F_n>D} \tag{7}$$

are the optimal control variables in climb. In descent, the functional is minimized and thrust is constrained to be less than drag.

In principle then, it should be relatively simple to minimize the climb, cruise, and descent functionals as the functionals are unimodal for a given altitude over the allowable aircraft speed and power settings. A difficulty lies in the shallowness of the functions. Quantization errors can lead to different solutions on different iteration cycles.

For example, consider the minimum fuel situation ($\sigma = 0$). In this case, it is more convenient to talk of maximizing specific range instead of performance. Figure 2 illustrates the specific range curves for a 727-200 for various aircraft weights at 35,000 ft. About the optimal speed, the speed can vary by ±0.03-0.05 Mach and still be within 1% of optimum. A numerical search algorithm may select any number of speeds within this range as the optimum speed. The problem is that if the search algorithm generates an optimal speed time history which is oscillatory, the engines accelerate and decelerate wasting fuel, reducing engine life, and resulting in poor ride quality. Hence, even if identical steady-state performance can be achieved with different speeds, it is imperative that speed stability be insured by some means. A criterion used was that the higher speed was always chosen for "identical" (within roundoff error) specific ranges.

Similar conditions occur in climb and descent. These functions are also unimodal at a given altitude. The curves are flatter at low altitudes than at high altitudes. The flatness of the curves is analogous to the specific range conditions illustrated earlier. Since the optimal speed at each altitude is not constant, it is more difficult to incorporate smoothing criteria on the climb and descent solutions than on the cruise solutions. The flight-path angle differential equation would give continuity to the optimal speed profiles if it were incorporated in the optimization. However, this would require a solution via the two-point boundary value problem which is not desirable because it would not allow real-time avionics computer implementation.

Recall that for the reduced problem formulation, flight-path dynamics are ignored in the development of the optimal controls strategy. The resultant functionals, which are minimized at each altitude, only depend on each other by the value of the cruise cost function at the top of climb or top of descent. The extremization of the shallow functionals at times results in significant flight-path angle changes for a small altitude change. See, for example, Table 8 in Ref. 6; when "optimally" descending through 26,108 ft the flight-path angle changes from −2.54 to −5.20 to −2.59 in about 650 ft with a similar aberration at 24,490 ft. Additional logic must be incorporated if these cyclic, energy-absorbing transients are to be avoided. Reference 11 presents a more detailed study on the functionals and control strategies.

Additional Constraints

Longitudinal speed restrictions (such as airframe limits or ATC limits) are required regardless of the control strategy chosen (optimal or conventional). Fortunately they are easily incorporated. If the desired longitudinal speed is not within

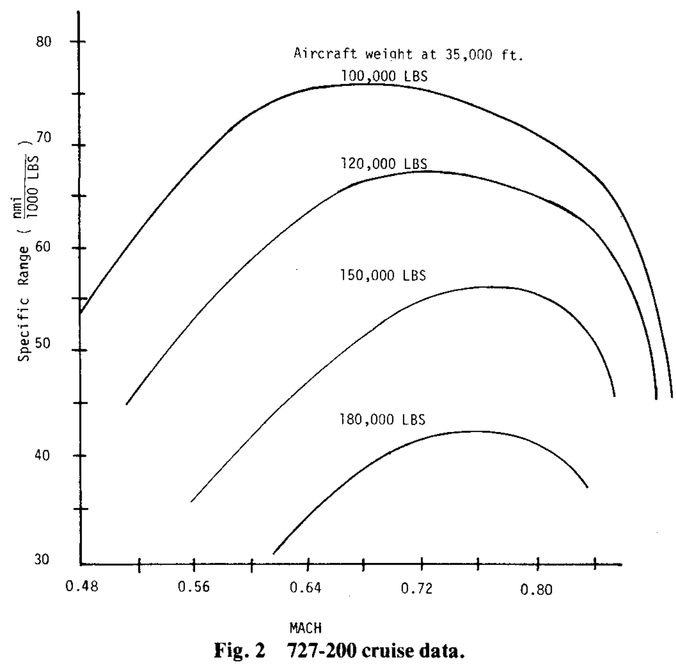

Fig. 2 727-200 cruise data.

the acceptable range of speeds, then the constrained longitudinal speed is on the speed restriction boundary closest to the desired speed (see Fig. 2).

Vertical speed restrictions must be incorporated into the trajectory integration and possibly thrust control search logic. Vertical speed effects occur indirectly by matching the control solutions at two altitudes. If thrust is constrained solely by the minimum and maximum engine limits, excessive vertical rates or flight-path angles can occur. These can be constrained during the search for optimum velocity and thrust by searching first on velocity and then limiting thrust to engine manufacturer constraints or the flight-path constraints, whichever is more limiting.

Altitude constraints fall into two categories: those that require a cruise altitude below the best operating altitude, and those that call for a cruise altitude above the best operating cruise altitude. The former situation will be treated first.

An assigned cruise altitude below an optimal cruise altitude can occur due to ATC or stage length limitations. For short stage lengths there may be insufficient range to climb to and descend from the optimal cruise altitude. For optimal flight control, since the theoretical development is concerned only with the conditions met at the cruise altitude, the optimal flight conditions are found when the altitude is constrained to the assigned cruise altitude and the optimal climb and descent conditions found. Should the stage length be too short, the altitude is decremented until a feasible solution is found. Hence, except for the need to determine climb and descent trajectories for a feasible solution, no computational problem exists.

There is no analogous optimal climb solution for an assigned cruise altitude above the optimal cruise altitude. An optimal climb trajectory above the optimal cruise altitude cannot be reached with the present problem formulation since the trajectory cannot pass through the singularity at the optimum altitude. As the aircraft approaches the optimal cruise altitude, the optimal solutions result in little or no excess thrust being applied with the vertical speed becoming essentially zero. In essence, the solution to the equations recognizes that this altitude should not be exceeded, based on the predicted performance criteria for the present optimal and the assigned cruise altitudes. To force a climb solution, a minimum climb rate (translated to a minimum excess thrust requirement) was imposed, but this is rather artificial depending on aircraft weight and maximum altitude. The most natural solution under these conditions was to determine the optimal cruise speed with thrust fixed at maximum climb thrust. The "cost" of using this or any approach restricted to climb is not measurable using the optimal control solution given by Eq. (5) because in the "optimal" solution the assigned cruise altitude is not reached. For climbs to altitudes below optimal cruise altitude, the cost of imposing maximum climb thrust was less than 1%.

Trajectory Integration

Trajectory integration along the lateral path using the optimal controls is necessary to define top of descent and to determine if the cruise altitude is feasible on short flights. Using the energy and altitude as state variables (with thrust and speed as control variables), direct trajectory integration is possible, whereas velocity altitude state variable formulation requires an iterative solution. The formulation is quite similar to that reported in Ref. 6. The major difference is in the energy formulation.

Two energy formulations were investigated: an air mass energy formulation and an inertial or ground-referenced formulation; the difference is use of true air speed in the former and ground speed in the latter. Climb and descent controls generated by either formulation yielded equivalent performance. Therefore, the computationally simpler air mass formulation[6] was used in the computation of the optimal controls. However, an inertial formulation is required for trajectory integration. This distinction manifests itself in the time between energy increments. The time between energy transitions, Δt, is defined in Ref. 6 as

$$\Delta t = \Delta E / \dot{E} \tag{8}$$

When energy is referenced to air mass, the effects of a changing wind are ignored. This results in an error in computing time between transitions, Δt, which corrupts subsequent computations. Conversion to an inertial reference in the trajectory computations eliminates this error.

Trajectory integration must distinguish between pressure altitude and true altitude. Most engine data are based on thrust and fuel flow conditions at a pressure altitude. Hence, it is appropriate to use pressure altitude when computing the optimal control speed and thrust settings. However, during trajectory integration an inertial reference frame should be used to compute the energy transitions. The optimization computations used here assume that the temperature model follows the standard atmosphere model with offset equal to the difference between measured temperature and the standard day temperature. The same is true for a pressure offset. True altitude is thus modeled by the temperature and pressure offsets. The remainder of the calculations were equivalent to those in Ref. 6.

Conclusions

Real-time optimization was shown to be feasible for determining optimal climb, cruise, and descent flight controls. However, the practical savings were marginal when compared to the perfectly flown handbook. In most instances, the climb and descent trajectories obtained from real-time optimization were better than IAS/Mach schedules. However, at times the trajectories were more costly than the IAS/Mach schedules due to the shallowness of the functions to be minimized and the limitations of the reduced order problem formulation. This was never the case in cruise. It is concluded that a prestored climb and descent speed schedule yields not only equivalent performance but also provides an inherent continuity in the schedule that is sometimes lacking in the real-time optimization for climb and descent. The prestored climb and descent speed schedules would depend on gross weight and altitude as a minimum. In descent, a dependence on cost is also required. The climb and descent optimization algorithms are still valuable as a performance reference to evaluate the prestored schedules. Real-time optimization is easier to justify for the cruise segment where for long flights, a small percentage improvement yields a significant fuel savings.

Regardless of the flight control strategy, knowledge of the geographic trajectory is required for flight planning purposes, for ATC advisories, and, in particular, for descent guidance. All savings can be lost through early descent. ATC may allow more discretionary descents if altitude crossing requirements are guaranteed to be met. Therefore, an accurately computed descent altitude vs range profile is necessary with the flight controls used to maintain the geographic descent profile.

The geographic descent profile can be prestored. However, the prestored descent profile can be quite complex due to the dimensionality of the problem which depends on altitude, weight, speed schedule, and temperature and wind profiles. Hence, it may be simpler to integrate the equations of motion in the onboard computer.

References

[1]Rutowski, E.S., "Energy Approach to the General Aircraft Performance Problem," *Journal of Aeronautical Sciences*, Vol. 21, March 1954, pp. 187-195.

[2]Thomas, A.N. and Porter, J.L., "Results of an Investigation of Propulsion Management Systems for Advanced Military Aircraft,"

Air Force Flight Dynamics Laboratory Tech. Rept. AFFDL-TR-71-136, Oct. 1971.

[3]Schultz, R.L. and Zagalsky, N.R., "Aircraft Performance Optimization," *Journal of Aircraft,* Vol. 9, Feb. 1972, pp. 108-114.

[4]Stengel, R.F. and Marcus, F.J., "Energy Management Techniques for Fuel Conservation in Military Transport Aircraft," Air Force Flight Dynamics Laboratory Tech. Rept. AFFDL-TR-75-156, Feb. 1976.

[5]Aggarwal, R., Calise, A.J. and Goldstein, F., "Singular Perturbation Analysis of Optimal Flight Profiles for Transport Aircraft," Paper TP-26, *Proceedings of the Joint Automatic Control Conference,* San Francisco, July 1977, pp. 1261-1269.

[6]Lee, H.Q. and Erzberger, H., "Algorithm for Fixed-Range Optimal Trajectories," NASA Tech. Paper 1565, July 1980.

[7]Erzberger, H. and Lee, H., "Constrained Optimum Trajectories with Specified Range," *Journal of Guidance and Control,* Vol. 3, Jan.-Feb. 1980, pp. 78-85.

[8]Bochem, J.H. and Mossman, D.C., "Simulator Evaluation of Optimal Thrust Management/Fuel Conservation Strategies for Airbus Aircraft on Short Haul Routes," NASA Contractor Rept. NAS 2-9174, June 1978.

[9]Marsh, J., "Climb Speed for Fuel Economy 707, 727, 737," *Boeing Airliner,* April 1978, pp. 3-12.

[10]Barman, J. and Erzberger, H., "Fixed Range Optimization Trajectories for Short Haul Aircraft," *Journal of Aircraft,* Vol. 13, Oct. 1976, pp. 748-754.

[11]Bruckner, J.M.H. and Sadowsky, J.S., "Flight Control Strategies for Performance Computers," *Navigation,* Vol. 27, Summer 1980, pp. 4-14.